

# Influence of energy exchange of electrons and ions on the long-wavelength thermal instability in magnetized astrophysical objects

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## ABSTRACT

We investigate thermal instability in an electron-ion magnetized plasma relevant to galaxy clusters, solar corona, and other two-component astrophysical objects. We apply the multicomponent plasma approach when the dynamics of all the species are considered separately through electric field perturbations. General expressions for perturbations obtained in this paper can be applied for a wide range of multicomponent astrophysical and laboratory plasmas also containing the neutrals, dust grains, and other species. We assume that background temperatures of electrons and ions are different and include the energy exchange in thermal equations. We take into account the dependence of collision frequency on density and temperature perturbations. The cooling-heating functions are taken as different ones for electrons and ions. As a specific case, we consider a condensation mode of thermal instability of long-wavelength perturbations when the dynamical time is smaller than a time during which the particles cover the wavelength along the magnetic field due to thermal velocity. We derive a general dispersion relation taking into account the effects mentioned above and obtain simple expressions for growth rates in limiting cases. Perturbations are shown to have an electromagnetic nature. We find that at conditions under consideration transverse scale sizes of unstable perturbations can have a wide

spectrum relatively to longitudinal scale sizes and, in particular, form very thin filaments. The results obtained can be useful for interpretation of observations of dense cold regions in astrophysical objects.

**Key words:** conduction – galaxies: clusters: general – instabilities – magnetic fields – plasmas – waves

## 1. INTRODUCTION

The thermal instability leads to formation of regions with larger densities and lower temperatures than that in the surrounding medium (Parker 1953; Field 1965). Beginning from the classical paper by Field (1965), this instability was studied for both astrophysical objects (for reviews see, e.g., Vázquez-Semadeni et al. 2003; Elmegreen & Scalo 2004; Cox 2005; Heiles & Crutcher 2005) and plasma physics applications (e.g., Meerson 1996). Majority of papers were devoted to thermal instability in the interstellar medium (ISM; e.g., Field 1965; Burkert & Lin 2000; Hennebelle & Péroult 2000; Koyama & Inutsuka 2002; Kritsuk & Norman 2002; Sánchez-Salcedo et al. 2002; Audit & Hennebelle 2005; Stiele et al. 2006; Vázquez-Semadeni et al. 2006; Fukue & Kamaya 2007; Inoue & Inutsuka 2008; Shadmehri et al. 2010). Solar prominences are supposed to be formed as a result of thermal instability (e.g., Field 1965; Nakagawa, 1970; Heyvaerts 1974; Mason & Bessey 1983; Karpen et al. 1989). In galaxy clusters, this instability, including the presence of the magnetic field, was studied in (e.g., Field 1965; Loewenstein 1990; Balbus 1991; Bogdanović et al. 2009; Parrish et al. 2009; Sharma et al. 2010). The nonlinear stage of thermal instability resulting in formation of nonlinear cool structures was investigated in the ISM (e.g., Trevisan & Ibáñez 2000; Sánchez-Salcedo et al. 2002; Yatou & Toh 2009) and solar corona (Mason & Bessey 1983; Karpen et al. 1989; Trevisan & Ibáñez 2000).

In papers studying thermal instability in astrophysical objects with the magnetic field, the one-fluid ideal MHD is generally used. The two-fluid model of the ideal MHD has been treated, e.g., by Fukue & Kamaya (2007) and Inoue & Inutsuka (2008). The non-ideal effects in the magnetic induction equation have been considered by several authors (e.g., Heyvaerts 1974; Stiele et al. 2006; Shadmehri et al. 2010).

For astrophysical media consisting of many kinds of species (electrons, ions, dust grains, neutrals, and so on), the multicomponent approach considering the dynamics of

each species separately is an adequate method of investigation (e.g., Nekrasov 2009a, 2009b, 2009c). The thermal instability in multicomponent media has been studied by Kopp et al. (1997), Pandey & Krishan (2001), Pandey et al. (2003), Shukla & Sandberg (2003), Kopp & Shchekinov (2007). Analytical investigation of thermal instability in multicomponent magnetized media with such physical effects as collisions between different species, ionization and recombination, dust charge dynamics, gravity, self-gravity, and so on is a sufficiently difficult problem. Therefore, one usually treats simplified models such as, for example, potential perturbations in nonmagnetized (Kopp et al. 1997; Pandey & Krishan 2001; Ibáñez & Shchekinov 2002; Pandey et al. 2003; Shukla & Sandberg 2003; Kopp & Shchekinov 2007) and magnetized (Kopp et al. 1997; Shukla & Sandberg 2003) plasmas.

When studying thermal instability, one usually does not take into account an energy exchange between species in thermal equations. It may be done at a weak or strong collisional coupling of species. However, an intermediate case can in general also occur. The inclusion of this effect results in considerable analytical complications (e.g., Birk 2000; Birk & Wiechen 2001). The absence of thermodynamical equilibrium is an additional factor complicating a problem. However, different temperatures of species can be observed, for example, in galaxy clusters (Markevitch et al. 1996; Fox & Loeb 1997; Ettori & Fabian 1998; Takizawa 1998). Therefore, this effect needs also to be taken into consideration. When background temperatures of species are different, it is necessary to take into account the perturbation of the energy exchange frequency which depends on the number density and temperature.

In this paper, thermal instability in the electron-ion magnetized plasma relevant to galaxy clusters, solar corona, and other two-component astrophysical objects is investigated. We apply the multicomponent plasma approach when the dynamics of all the species

are considered separately through electric field perturbations (the **E**-approach; see, e.g., Nekrasov 2009a, 2009b, 2009c; Nekrasov & Shadmehri 2010, 2011). General expressions obtained in this paper can be applied for a wide range of astrophysical and laboratory plasmas also containing the neutrals and dust grains. We assume that background temperatures of electrons and ions are different and include the energy exchange in thermal equations. We take into account the dependence of energy exchange collision frequency on density and temperature perturbations. The cooling-heating functions are also considered for both electrons and ions. We do not include ionization and recombination effects and the gravity. Expressions for electron and ion perturbations are obtained in the general form which can be used for other species. As a specific case, we here treat a condensation mode of thermal instability of perturbations elongated enough along the background magnetic field. In this case, the dynamical time is smaller than a time during of which the particles cover the longitudinal wavelength due to their thermal velocity. The opposite, fast sound speed limit, is considered in (Nekrasov 2011). We derive the general dispersion relation, taking into account the effects mentioned above, and discuss the limiting cases.

The paper is organized in the following manner. In Section 2, we give fundamental equations used in this paper. An equilibrium state is considered in Section 3. General equations for temperature perturbations are obtained in Section 4. In Section 5, equations for components of velocity perturbations are given in the fast dynamical regime. Components of perturbed current are calculated in Section 6. These components for the simplified collision contribution are given in Section 7. In Section 8, we derive the dispersion relation. Its limiting cases are considered in Section 9. We discuss the obtained results in Section 10. The possible astrophysical implications are considered in Section 11. Summing up of main points is given in Section 12.

## 2. BASIC EQUATIONS

The fundamental equations we use are

$$\frac{\partial \mathbf{v}_j}{\partial t} + \mathbf{v}_j \cdot \nabla \mathbf{v}_j = -\frac{\nabla p_j}{m_j n_j} + \mathbf{F}_j + \frac{q_j}{m_j c} \mathbf{v}_j \times \mathbf{B}, \quad (1)$$

the equation of motion,

$$\frac{\partial n_j}{\partial t} + \nabla \cdot n_j \mathbf{v}_j = 0, \quad (2)$$

the continuity equation,

$$\frac{\partial T_i}{\partial t} + \mathbf{v}_i \cdot \nabla T_i + (\gamma - 1) T_i \nabla \cdot \mathbf{v}_i = -(\gamma - 1) \frac{1}{n_i} \mathcal{L}_i(n_i, T_i) + \nu_{ie}^\varepsilon(n_e, T_e)(T_e - T_i) \quad (3)$$

and

$$\frac{\partial T_e}{\partial t} + \mathbf{v}_e \cdot \nabla T_e + (\gamma - 1) T_e \nabla \cdot \mathbf{v}_e = -(\gamma - 1) \frac{1}{n_e} \nabla \cdot \mathbf{q}_e - (\gamma - 1) \frac{1}{n_e} \mathcal{L}_e(n_e, T_e) - \nu_{ei}^\varepsilon(n_i, T_e)(T_e - T_i) \quad (4)$$

are temperature equations for ions and electrons. In Equations (1) and (2), the index  $j = i, e$  denotes ions and electrons, respectively. The value  $\mathbf{F}_j$  in Equation (1) is given by

$$\begin{aligned} \mathbf{F}_i &= \frac{q_i}{m_i} \mathbf{E} - \nu_{ie}(\mathbf{v}_i - \mathbf{v}_e), \\ \mathbf{F}_e &= \frac{q_e}{m_e} \mathbf{E} - \nu_{ei}(\mathbf{v}_e - \mathbf{v}_i). \end{aligned} \quad (5)$$

Other notations in Equations (1)-(5) are the following:  $q_j$  and  $m_j$  are the charge and mass of species  $j = i, e$ ,  $\mathbf{v}_j$  is the hydrodynamic velocity,  $n_j$  is the number density,  $p_j = n_j T_j$  is the thermal pressure,  $T_j$  is the temperature in the energy units,  $\nu_{ie}$  ( $\nu_{ei}$ ) is the collision frequency of ions (electrons) with electrons (ions),  $\nu_{ie}^\varepsilon(n_e, T_e) = 2\nu_{ie}(\nu_{ei}^\varepsilon(n_i, T_e))$  is the frequency of the thermal energy exchange between ions (electrons) and electrons (ions) (Braginskii 1965),  $n_i \nu_{ie}^\varepsilon(n_e, T_e) = n_e \nu_{ei}^\varepsilon(n_i, T_e)$ ,  $\gamma$  is the ratio of specific heats,  $\mathbf{E}$  and  $\mathbf{B}$  are the electric and magnetic fields, and  $c$  is the speed of light in vacuum. The value  $\mathbf{q}_e$

in Equation (4) is the electron heat flux associated with the thermal motion in the system of coordinates where the electron gas is at rest as a whole (Braginskii 1965). As for the latter, we will consider a weakly collisional plasma when the electron Larmor radius is much smaller than the electron collisional mean free path. In this case, the electron heat flux is mainly directed along the magnetic field,

$$\mathbf{q}_e = -\chi_e \mathbf{b} (\mathbf{b} \cdot \nabla) T_e, \quad (6)$$

where  $\chi_e$  is the electron thermal conductivity coefficient and  $\mathbf{b} = \mathbf{B}/B$  is the unit vector along the magnetic field. In other respects, a relation between cyclotron and collision frequencies of species stays arbitrary in general expressions considered below. We only take into account the electron heat flux (6) because the corresponding ion thermal conductivity is considerably smaller (Braginskii 1965). We also assume that the heat flux in equilibrium is absent. The cooling and heating of plasma species in Equations (3) and (4) are described by function  $\mathcal{L}_j(n_j, T_j) = n_j^2 \Lambda_j(T_j) - n_j \Gamma_j$ , where  $\Lambda_j$  and  $\Gamma_j$  are the cooling and heating functions, respectively. The form of this function differs from the usually used cooling-heating function  $\mathcal{L}$ , beginning from the classic paper by Field (1965). Both functions are connected with each other via equality  $\mathcal{L}_j(n_j, T_j) = m_j n_j \mathcal{L}_j$ . Our choice is analogous to that as in Begelman & Zweibel (1994), Pandey & Krishan (2001), Shukla & Sandberg (2003), Bogdanović et al. (2009), Parrish et al. (2009). The function  $\Lambda_j(T_j)$  can be found, for example, in Tozzi & Norman (2001).

Electromagnetic equations are Faraday's

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \quad (7)$$

and Ampere's

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j} \quad (8)$$

laws, where  $\mathbf{j} = \sum_j q_j n_j \mathbf{v}_j$ . We consider wave processes with typical timescales much larger than the time the light spends to cover the wavelength of perturbations. In this case, one

can neglect the displacement current in Equation (8) that results in quasi-neutrality for both electromagnetic and purely electrostatic perturbations. The magnetic field  $\mathbf{B}$  includes the background magnetic field  $\mathbf{B}_0$ , the magnetic field  $\mathbf{B}_{0cur}$  of the background electric current (when is present), and the perturbed magnetic field.

For generality, we assume in the meanwhile that  $n_i \neq n_e$ , having in mind that some expressions obtained below can be applied for multicomponent plasmas.

### 3. EQUILIBRIUM STATE

At first, we will consider an equilibrium state. We assume that the background flow (average) velocities of species are absent. We do not here involve an equilibrium inhomogeneity. Then, thermal equations (3) and (4) in equilibrium take the form

$$\begin{aligned} (\gamma - 1) \frac{1}{n_{i0}} \mathcal{L}_i(n_{i0}, T_{i0}) - \nu_{ie}^\varepsilon(n_{e0}, T_{e0})(T_{e0} - T_{i0}) &= 0, \\ (\gamma - 1) \frac{1}{n_{e0}} \mathcal{L}_e(n_{e0}, T_{e0}) + \nu_{ei}^\varepsilon(n_{i0}, T_{e0})(T_{e0} - T_{i0}) &= 0, \end{aligned} \quad (9)$$

where the subscript 0 denotes equilibrium values.

### 4. LINEAR EQUATIONS FOR TEMPERATURE PERTURBATIONS

We now consider Equations (3) and (4) in the linear approximation. Applying the operator  $\partial/\partial t$  to Equation (3) and using for ions Equation (2) to exclude the number density perturbation and Equation (9), we find

$$D_{1i}T_{i1} - D_{2i}T_{e1} = C_{1i}\nabla \cdot \mathbf{v}_{i1} - C_{2i}\nabla \cdot \mathbf{v}_{e1}, \quad (10)$$



where and below the subscript 1 denotes perturbed values. The operators and notations introduced in Equation (10) are as follows:

$$\begin{aligned}
D_{1i} &= \left( \frac{\partial}{\partial t} + \Omega_{Ti} + \Omega_{ie} \right) \frac{\partial}{\partial t}, \\
D_{2i} &= (\Omega_{Tie} + \Omega_{ie}) \frac{\partial}{\partial t}, \\
C_{1i} &= T_{i0} \left[ -(\gamma - 1) \frac{\partial}{\partial t} + \Omega_{ni} - \frac{(T_{e0} - T_{i0})}{T_{i0}} \Omega_{ie} \right], \\
C_{2i} &= \Omega_{ie} (T_{e0} - T_{i0}).
\end{aligned} \tag{11}$$

Analogously, we obtain for electrons

$$D_{1e} T_{e1} - D_{2e} T_{i1} = C_{1e} \nabla \cdot \mathbf{v}_{e1} + C_{2e} \nabla \cdot \mathbf{v}_{i1}, \tag{12}$$

where

$$\begin{aligned}
D_{1e} &= \left( \frac{\partial}{\partial t} + \Omega_{\chi} + \Omega_{Te} + \Omega_{Tei} + \Omega_{ei} \right) \frac{\partial}{\partial t}, \\
D_{2e} &= \Omega_{ei} \frac{\partial}{\partial t}, \\
C_{1e} &= T_{e0} \left[ -(\gamma - 1) \frac{\partial}{\partial t} + \Omega_{ne} + \frac{(T_{e0} - T_{i0})}{T_{e0}} \Omega_{ei} \right], \\
C_{2e} &= \Omega_{ei} (T_{e0} - T_{i0}).
\end{aligned} \tag{13}$$

In notations (11) and (13), we have introduced the following frequencies:

$$\begin{aligned}
\Omega_{\chi} &= -(\gamma - 1) \frac{\chi_{e0}}{n_{e0}} \frac{\partial^2}{\partial z^2}, \\
\Omega_{Te} &= (\gamma - 1) \frac{\partial \mathcal{L}_e(n_{e0}, T_{e0})}{n_{e0} \partial T_{e0}}, \Omega_{Ti} = (\gamma - 1) \frac{\partial \mathcal{L}_i(n_{i0}, T_{i0})}{n_{i0} \partial T_{i0}}, \\
\Omega_{ne} &= (\gamma - 1) \frac{\partial \mathcal{L}_e(n_{e0}, T_{e0})}{T_{e0} \partial n_{e0}}, \Omega_{ni} = (\gamma - 1) \frac{\partial \mathcal{L}_i(n_{i0}, T_{i0})}{T_{i0} \partial n_{i0}}, \\
\Omega_{ei} &= \nu_{ei}^{\varepsilon}(n_{i0}, T_{e0}), \Omega_{ie} = \nu_{ie}^{\varepsilon}(n_{e0}, T_{e0}), \\
\Omega_{Tei} &= \frac{\partial \nu_{ei}^{\varepsilon}(n_{i0}, T_{e0})}{\partial T_{e0}} (T_{e0} - T_{i0}), \Omega_{Tie} = \frac{\partial \nu_{ie}^{\varepsilon}(n_{e0}, T_{e0})}{\partial T_{e0}} (T_{e0} - T_{i0}).
\end{aligned} \tag{14}$$

We assume that the background magnetic field  $\mathbf{B}_0$  is directed along the  $z$ -axis. In notations (11) and (13), we have used the equilibrium state and the number density dependence of  $\nu_{ei}^\varepsilon(n_{i0}, T_{e0}) \sim n_{i0}$  and  $\nu_{ie}^\varepsilon(n_{e0}, T_{e0}) \sim n_{e0}$ . We see from Equations (10) and (12) that temperature perturbations are connected with a velocity divergence. Solutions for  $T_{e1}$  and  $T_{i1}$  are given by

$$DT_{e1} = G_1 \nabla \cdot \mathbf{v}_{e1} + G_2 \nabla \cdot \mathbf{v}_{i1}, \quad (15)$$

$$DT_{i1} = G_3 \nabla \cdot \mathbf{v}_{e1} + G_4 \nabla \cdot \mathbf{v}_{i1}, \quad (16)$$

where the following notations are introduced:

$$D = (D_{1i}D_{1e} - D_{2i}D_{2e}), \quad (17)$$

$$G_1 = (D_{1i}C_{1e} - D_{2e}C_{2i}),$$

$$G_2 = (D_{1i}C_{2e} + D_{2e}C_{1i}),$$

$$G_3 = (D_{2i}C_{1e} - D_{1e}C_{2i}),$$

$$G_4 = (D_{1e}C_{1i} + D_{2i}C_{2e}).$$

To find the temperature perturbation  $T_{j1}$ , we must have expressions for  $\nabla \cdot \mathbf{v}_{j1}$ . General equations for the velocity  $\mathbf{v}_{j1}$  and  $\nabla \cdot \mathbf{v}_{j1}$  are derived in the Appendix, where expressions for  $D$  and  $G_l$ ,  $l = 1, 2, 3, 4$ , are also given. In their general form, the components of  $\mathbf{v}_{j1}$  are very complex. Therefore to proceed further analytically, we here restrict ourselves to a limiting case in which the dynamical time  $(\partial/\partial t)^{-1}$  is short in comparison with a time the thermal particles need to cover the wavelength along the magnetic field. Some additional simplifying conditions which are satisfied in magnetized plasmas are also used.

We note that the opposite case when the dynamical frequency is smaller then the corresponding sound frequency has been considered in (Nekrasov 2011). The general expressions are the same in the last and this papers. However for convenience of reading, we keep their here.

### 5. SPECIFIC CASE: $\frac{\partial^2}{\partial t^2} \gg \frac{\partial^2}{\partial z^2} (v_{Te}^2 + v_{Ti}^2)$

Equations (A27), (A28) and (A30) are written in their general form, which allows us to consider different simplified specific cases corresponding to real astrophysical conditions. We further proceed with sufficiently fast perturbations such that

$$\frac{\partial^2}{\partial t^2} \gg \frac{\partial^2}{\partial z^2} (v_{Te}^2 + v_{Ti}^2). \quad (18)$$

This condition is opposite to the one for the fast sound regime (Nekrasov 2011) and corresponds to the long-wavelength perturbations along the magnetic field. At the same time, we assume that  $\omega_{ci}^2 \gg \partial^2/\partial t^2$  for magnetized plasma. Other condition is a common one for hydromagnetic description, i.e.

$$1 \gg \frac{\partial^2}{\partial y^2} \left( \frac{v_{Te}^2}{\omega_{ce}^2} + \frac{v_{Ti}^2}{\omega_{ci}^2} \right), \quad (19)$$

when the Larmor radius of species is much smaller than the transverse wavelength of perturbations. The square of the velocity  $v_{Tj}^2$  has the following estimation:

$$\begin{aligned} v_{Te}^2 &= \frac{T_{e0} \left( \frac{\partial}{\partial t} + \Omega_{ie} \right) + T_{i0} \Omega_{ei}}{m_e \left( \frac{\partial}{\partial t} + \Omega_{ie} + \Omega_{ei} \right)}, \\ v_{Ti}^2 &= \frac{T_{i0} \left( \frac{\partial}{\partial t} + \Omega_{ei} \right) + T_{e0} \Omega_{ie}}{m_i \left( \frac{\partial}{\partial t} + \Omega_{ie} + \Omega_{ei} \right)}. \end{aligned} \quad (20)$$

Expressions (20) are given in the approximate form to unite two cases,  $\frac{\partial}{\partial t} > (\text{or } <) \Omega_{ie,ei}$ . We note that operators  $(\partial/\partial t)^{-1}$  and  $(\partial/\partial \mathbf{r})^{-1}$  in expressions (18)-(20) and corresponding expressions below denote typical dynamical times and wavelengths of perturbations. Under conditions (18) and (19) and using notations (A10), (A30), and (A31), we find equations for  $P_{i,e1}$  (see Equations (A27) and (A28)):

$$P_{i1} = \lambda_i \left( -\frac{1}{\omega_{ci}} \frac{\partial^2 F_{i1x}}{\partial y \partial t} + \frac{\partial F_{i1z}}{\partial z} \right) - \mu_i \left( -\frac{1}{\omega_{ce}} \frac{\partial^2 F_{e1x}}{\partial y \partial t} + \frac{\partial F_{e1z}}{\partial z} \right), \quad (21)$$

$$P_{e1} = \lambda_e \left( -\frac{1}{\omega_{ce}} \frac{\partial^2 F_{e1x}}{\partial y \partial t} + \frac{\partial F_{e1z}}{\partial z} \right) - \mu_e \left( -\frac{1}{\omega_{ci}} \frac{\partial^2 F_{i1x}}{\partial y \partial t} + \frac{\partial F_{i1z}}{\partial z} \right), \quad (22)$$

where the following notations are introduced:

$$\begin{aligned}\lambda_i &= \frac{T_{i0}}{m_i} \left( \frac{\partial}{\partial t} \right)^{-1} - \frac{G_4}{Dm_i}, \mu_i = \frac{G_3}{Dm_i}, \\ \lambda_e &= \frac{T_{e0}}{m_e} \left( \frac{\partial}{\partial t} \right)^{-1} - \frac{G_1}{Dm_e}, \mu_e = \frac{G_2}{Dm_e}.\end{aligned}\tag{23}$$

Equations (21) and (22) have a symmetric form relatively to changing the index  $i$  by  $e$  and vice versa. Estimations of  $\lambda_j$  and  $\mu_j$  are followed from expressions (A16), (A18), (A19), and (A29).

### 5.1. EQUATIONS FOR COMPONENTS OF VELOCITIES $\mathbf{v}_{i,e1}$

We now obtain equations for components of velocities  $\mathbf{v}_{i,e1}$ , using Equations (21) and (22).

#### 5.1.1. Equations for $v_{i,e1y}$

From Equations (A3), (21), and (22), we find, using notations (A6),

$$\begin{aligned}v_{i1y} &= -\frac{1}{\omega_{ci}} F_{i1x} + \frac{1}{\omega_{ci}^3} \frac{\partial^2 F_{i1x}}{\partial t^2} + \frac{1}{\omega_{ci}^2} \frac{\partial F_{i1y}}{\partial t} - \frac{1}{\omega_{ci}^4} \frac{\partial^3 F_{i1y}}{\partial t^3} \\ &\quad - \frac{1}{\omega_{ci}^2} \frac{\partial^3}{\partial y^2 \partial t} \left( \frac{\lambda_i}{\omega_{ci}} F_{i1x} - \frac{\mu_i}{\omega_{ce}} F_{e1x} \right) \\ &\quad + \frac{1}{\omega_{ci}^2} \frac{\partial^2}{\partial y \partial z} (\lambda_i F_{i1z} - \mu_i F_{e1z}),\end{aligned}\tag{24}$$

$$\begin{aligned}v_{e1y} &= -\frac{1}{\omega_{ce}} F_{e1x} + \frac{1}{\omega_{ce}^3} \frac{\partial^2 F_{e1x}}{\partial t^2} + \frac{1}{\omega_{ce}^2} \frac{\partial F_{e1y}}{\partial t} - \frac{1}{\omega_{ce}^4} \frac{\partial^3 F_{e1y}}{\partial t^3} \\ &\quad - \frac{1}{\omega_{ce}^2} \frac{\partial^3}{\partial y^2 \partial t} \left( \frac{\lambda_e}{\omega_{ce}} F_{e1x} - \frac{\mu_e}{\omega_{ci}} F_{i1x} \right) \\ &\quad + \frac{1}{\omega_{ce}^2} \frac{\partial^2}{\partial y \partial z} (\lambda_e F_{e1z} - \mu_e F_{i1z}).\end{aligned}\tag{25}$$

The terms proportional to  $\omega_{cj}^{-4}$  are needed in equations for  $v_{i,e1x}$  for obtaining terms  $\sim \omega_{ej}^{-3}$ .

We see that these solutions are obtained one from another by changing  $i \leftrightarrow e$ .

### 5.1.2. Equations for $v_{i,e1x}$

Equations for  $v_{i,e1x}$  are easily found from Equation (A2) by using Equations (24) and (25):

$$\begin{aligned} v_{i1x} = & \frac{1}{\omega_{ci}} F_{i1y} + \frac{1}{\omega_{ci}^2} \frac{\partial F_{i1x}}{\partial t} - \frac{1}{\omega_{ci}^3} \frac{\partial^2 F_{i1y}}{\partial t^2} \\ & - \frac{1}{\omega_{ci}} \frac{\partial^2}{\partial y^2} \left( \frac{\lambda_i}{\omega_{ci}} F_{i1x} - \frac{\mu_i}{\omega_{ce}} F_{e1x} \right) \\ & + \frac{1}{\omega_{ci}} \frac{\partial^2}{\partial y \partial z} \left( \frac{\partial}{\partial t} \right)^{-1} (\lambda_i F_{i1z} - \mu_i F_{e1z}), \end{aligned} \quad (26)$$

$$\begin{aligned} v_{e1x} = & \frac{1}{\omega_{ce}} F_{e1y} + \frac{1}{\omega_{ce}^2} \frac{\partial F_{e1x}}{\partial t} - \frac{1}{\omega_{ce}^3} \frac{\partial^2 F_{e1y}}{\partial t^2} \\ & - \frac{1}{\omega_{ce}} \frac{\partial^2}{\partial y^2} \left( \frac{\lambda_e}{\omega_{ce}} F_{e1x} - \frac{\mu_e}{\omega_{ci}} F_{i1x} \right) \\ & + \frac{1}{\omega_{ce}} \frac{\partial^2}{\partial y \partial z} \left( \frac{\partial}{\partial t} \right)^{-1} (\lambda_e F_{e1z} - \mu_e F_{i1z}). \end{aligned} \quad (27)$$

### 5.1.3. Equations for $v_{i,e1z}$

From Equations (A7), (21), and (22), we obtain, using notations (A6),

$$\begin{aligned} \frac{\partial^2 v_{i1z}}{\partial t^2} = & \frac{\partial F_{i1z}}{\partial t} - \frac{\partial^3}{\partial y \partial z \partial t} \left( \frac{\lambda_i}{\omega_{ci}} F_{i1x} - \frac{\mu_i}{\omega_{ce}} F_{e1x} \right) \\ & + \frac{\partial^2}{\partial z^2} (\lambda_i F_{i1z} - \mu_i F_{e1z}), \end{aligned} \quad (28)$$

$$\begin{aligned} \frac{\partial^2 v_{e1z}}{\partial t^2} = & \frac{\partial F_{e1z}}{\partial t} - \frac{\partial^3}{\partial y \partial z \partial t} \left( \frac{\lambda_e}{\omega_{ce}} F_{e1x} - \frac{\mu_e}{\omega_{ci}} F_{i1x} \right) \\ & + \frac{\partial^2}{\partial z^2} (\lambda_e F_{e1z} - \mu_e F_{i1z}). \end{aligned} \quad (29)$$

## 6. COMPONENTS OF CURRENT

We now find components of the linear current  $\mathbf{j}_1 = \sum_j q_j n_{j0} \mathbf{v}_{j1}$ . It is convenient to consider the value  $4\pi (\partial/\partial t)^{-1} \mathbf{j}_1$ . In this case, we obtain dimensionless coefficients by  $\mathbf{E}_1$ . We further consider the electron-ion plasma in which  $n_{e0} = n_{i0} = n_0$ ,  $q_e = -q_i$ . In our calculations, we will use an equality  $m_e \nu_{ei} = m_i \nu_{ie}$  and the relation  $m_i \mathbf{F}_{i1} = -m_e \mathbf{F}_{e1}$ . From Equations (24)-(29) and using notations (5) in the linear approximation, we find

$$4\pi \left( \frac{\partial}{\partial t} \right)^{-1} j_{1x} = a_{xx} E_{1x} - a_{xy} E_{1y} + a_{xz} E_{1z} \quad (30)$$

$$- b_{xx} (v_{i1x} - v_{e1x}) + b_{xy} (v_{i1y} - v_{e1y}) - b_{xz} (v_{i1z} - v_{e1z}),$$

$$4\pi \left( \frac{\partial}{\partial t} \right)^{-1} j_{1y} = a_{yx} E_{1x} + a_{yy} E_{1y} + a_{yz} E_{1z} \quad (31)$$

$$- b_{yx} (v_{i1x} - v_{e1x}) - b_{yy} (v_{i1y} - v_{e1y}) - b_{yz} (v_{i1z} - v_{e1z}),$$

$$4\pi \left( \frac{\partial}{\partial t} \right)^{-1} j_{1z} = a_{zx} E_{1x} + a_{zz} E_{1z} \quad (32)$$

$$- b_{zx} (v_{i1x} - v_{e1x}) - b_{zz} (v_{i1z} - v_{e1z}),$$

Here, the following notations are introduced:

$$a_{xx} = \frac{\omega_{pi}^2}{\omega_{ci}^2} \left\{ 1 - \frac{1}{m_i} [(\lambda_e - \mu_e) m_e + (\lambda_i - \mu_i) m_i] \frac{\partial^2}{\partial y^2} \left( \frac{\partial}{\partial t} \right)^{-1} \right\}, \quad (33)$$

$$a_{xy} = \frac{\omega_{pi}^2}{\omega_{ci}^3} \frac{\partial}{\partial t}, a_{xz} = \frac{\omega_{pi}^2}{\omega_{ci}} \left[ \frac{\lambda_i m_i - \mu_e m_e}{m_i} - \frac{\lambda_e m_e - \mu_i m_i}{m_e} \right] \frac{\partial^2}{\partial y \partial z} \left( \frac{\partial}{\partial t} \right)^{-2},$$

$$a_{yx} = \frac{\omega_{pi}^2}{\omega_{ci}^2} \left\{ \frac{1}{\omega_{ci}} \frac{\partial}{\partial t} - \frac{1}{m_i} \left[ \frac{(\lambda_i - \mu_i) m_i}{\omega_{ci}} + \frac{(\lambda_e - \mu_e) m_e}{\omega_{ce}} \right] \frac{\partial^2}{\partial y^2} \right\}, a_{yy} = \frac{\omega_{pi}^2}{\omega_{ci}^2},$$

$$a_{yz} = \frac{\omega_{pi}^2}{\omega_{ci}} \left[ \frac{1}{\omega_{ci}} \left( \lambda_i + \mu_i \frac{m_i}{m_e} \right) - \frac{1}{\omega_{ce}} \left( \lambda_e + \mu_e \frac{m_e}{m_i} \right) \right] \frac{\partial^2}{\partial y \partial z} \left( \frac{\partial}{\partial t} \right)^{-1},$$

$$a_{zx} = \frac{\omega_{pi}^2}{\omega_{ci}} [(\lambda_e - \mu_e) - (\lambda_i - \mu_i)] \frac{\partial^2}{\partial y \partial z} \left( \frac{\partial}{\partial t} \right)^{-2}, a_{zy} = 0, a_{zz} = \omega_{pi}^2 \frac{m_i}{m_e} \left( \frac{\partial}{\partial t} \right)^{-2},$$

$$b_{ij} = a_{ij} \frac{m_i}{q_i} \nu_{ie}.$$

where  $\omega_{pi} = (4\pi n_{i0} q_i^2 / m_i)^{1/2}$  is the ion plasma frequency.

## 7. SIMPLIFICATION OF COLLISION CONTRIBUTION

Relationship between  $\omega_{ce}$  and  $\nu_{ei}$  or  $\omega_{ci}$  and  $\nu_{ie}$  (that is the same) can be arbitrary in Equations (30)-(32) (except of that in the thermal conduction). We further proceed by taking into account that  $\partial/\partial t \ll \omega_{ci}$ . In this case, we can neglect collisional terms proportional to  $b_{xy}$  and  $b_{yx}$  (see notations (33)). However, a system of Equations (30)-(32) stays sufficiently complex to find  $\mathbf{j}_1$  through  $\mathbf{E}_1$ . Therefore, we further consider the case in which the following condition is satisfied:

$$1 \gg \frac{\nu_{ie}}{\omega_{ci}^2} \frac{\partial}{\partial t} \left\{ 1 - \frac{1}{m_i} [m_e (\lambda_e - \mu_e) + m_i (\lambda_i - \mu_i)] \frac{\partial^2}{\partial y^2} \left( \frac{\partial}{\partial t} \right)^{-1} \right\} \quad (34)$$

or on the order of magnitude

$$1 \gg \frac{\nu_{ie}}{\omega_{ci}^2} \frac{\partial}{\partial t} \left[ 1 + c_s^2 \frac{\partial^2}{\partial y^2} \left( \frac{\partial}{\partial t} \right)^{-2} \right],$$

where  $c_s^2 = (T_{e0} + T_{i0}) / m_i$ . It is obvious that this inequality can easily be realized in magnetized plasma. Under condition (34), we can neglect the terms  $\sim b_{xx}$  and  $b_{zx}$  in Equations (30) and (32). In the case  $\omega_{ci}^2 \gg \nu_{ie} \partial/\partial t$ , the term  $\sim b_{yy}$  in Equation (31) can also be omitted. Thus, a system of Equations (30)-(32) takes the form,

$$4\pi \left( \frac{\partial}{\partial t} \right)^{-1} j_{1x} = \varepsilon_{xx} E_{1x} - \varepsilon_{xy} E_{1y} + \varepsilon_{xz} E_{1z}, \quad (35)$$

$$4\pi \left( \frac{\partial}{\partial t} \right)^{-1} j_{1y} = \varepsilon_{yx} E_{1x} + \varepsilon_{yy} E_{1y} + \varepsilon_{yz} E_{1z}, \quad (36)$$

$$4\pi \left( \frac{\partial}{\partial t} \right)^{-1} j_{1z} = \varepsilon_{zx} E_{1x} + \varepsilon_{zz} E_{1z}. \quad (37)$$

The following notations are here introduced:

$$\begin{aligned}\varepsilon_{xx} &= a_{xx} - \frac{\nu_{ie}}{\omega_{pi}^2} \frac{\partial}{\partial t} \frac{a_{xz}a_{zx}}{(1+d)}, \varepsilon_{xy} = a_{xy}, \varepsilon_{xz} = \frac{a_{xz}}{(1+d)}, \\ \varepsilon_{yx} &= a_{yx} - \frac{\nu_{ie}}{\omega_{pi}^2} \frac{\partial}{\partial t} \frac{a_{yz}a_{zx}}{(1+d)}, \varepsilon_{yy} = a_{yy}, \varepsilon_{yz} = \frac{a_{yz}}{(1+d)}, \\ \varepsilon_{zx} &= \frac{a_{zx}}{(1+d)}, \varepsilon_{zz} = \frac{a_{zz}}{(1+d)},\end{aligned}\tag{38}$$

where

$$d = a_{zz} \frac{\nu_{ie}}{\omega_{pi}^2} \frac{\partial}{\partial t} = \nu_{ei} \left( \frac{\partial}{\partial t} \right)^{-1}.\tag{39}$$

Parameter  $d$  defines the collisionless,  $d \ll 1$ , and collisional,  $d \gg 1$ , regimes. Below, we derive the dispersion relation.

## 8. DISPERSION RELATION AND ELECTRIC FIELD POLARIZATION

We further consider Equations (35)-(37) in the Fourier-representation, assuming that perturbations have the form  $\exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t)$ . Then using Equations (7) and (8), we obtain the following system of equations:

$$\begin{aligned}(n^2 - \varepsilon_{xx}) E_{1xk} + \varepsilon_{xy} E_{1yk} - \varepsilon_{xz} E_{1zk} &= 0, \\ -\varepsilon_{yx} E_{1xk} + (n_z^2 - \varepsilon_{yy}) E_{1yk} - (n_y n_z + \varepsilon_{yz}) E_{1zk} &= 0, \\ -\varepsilon_{zx} E_{1xk} - n_y n_z E_{1yk} + (n_y^2 - \varepsilon_{zz}) E_{1zk} &= 0,\end{aligned}\tag{40}$$

where  $\mathbf{E}_{1k}$  is the Fourier-image of the electric field perturbation,  $\mathbf{n} = \mathbf{k}c/\omega$ . The index  $k$  by  $\mathbf{E}_{1k}$  is equal to  $k = \{\mathbf{k}, \omega\}$ . For the Fourier-images of operators  $\varepsilon_{ij}$  and  $d$ , we keep the same notations. In general, we see that the longitudinal electric field  $E_{1zk} \sim E_{1x,yk}$  inevitably arises when  $k_y \neq 0$  and  $n_y^2 - \varepsilon_{zz} \neq 0$ . The dispersion relation can be found by setting the determinant of the system (40) equal to zero.



We will consider the case in which

$$\varepsilon_{zz} \gg n_y^2. \quad (41)$$

In the collisionless regime ( $d \ll 1$ ), this inequality denotes that the transverse wavelength of perturbations is much larger than the electron skin-depth. We assume that condition (41) is also satisfied in the collisional regime ( $d \gg 1$ ). Expressing the electric field  $E_{1zk}$  through  $E_{1x,yk}$  from the third Equation (40) and substituting it into two other equations, we find, using expressions (33) and (38), that contribution of the longitudinal electric field  $E_{1zk}$  in the case (18) is negligible. For an estimation of values (33), we take

$$(\lambda_j - \mu_j) m_j \sim (T_{e0} + T_{i0}) \left( \frac{\partial}{\partial t} \right)^{-1} \quad (42)$$

(see expressions (23)). From Equations (33) and (38), it is easy to see that  $\varepsilon_{yx}/\varepsilon_{xx} \sim \omega/\omega_{ci}$  and  $\varepsilon_{xy}/\varepsilon_{yy} \sim \omega/\omega_{ci}$ . Since  $\omega^2 \ll \omega_{ci}^2$ , the contribution of term  $\varepsilon_{xy}\varepsilon_{yx}$  into dispersion relation is small as compared with that of term  $\varepsilon_{xx}\varepsilon_{yy}$ . Then using an estimation (42), we obtain the simple dispersion relation

$$(n_z^2 - \varepsilon_{yy}) (n^2 - \varepsilon_{xx}) = 0. \quad (43)$$

The first factor of Equation (43) describes the Alfvén wave  $\omega^2 = k_z^2 c_A^2$ , where  $c_A = B_0/(4\pi m_i n_{i0})^{1/2}$  is the Alfvén velocity. The polarization of this wave is  $\mathbf{E}_1 = (0, E_{1y}, 0)$ . The second factor of Equation (43),

$$n^2 - \varepsilon_{xx} = 0, \quad (44)$$

describes the magnetosonic kind of perturbations with polarization  $\mathbf{E}_1 = (E_{1x}, 0, 0)$ . This perturbation is purely the electromagnetic one.

## 9. SOLUTION OF DISPERSION RELATION (44)

The parameter  $d$  defined by Equation (39) is equal to  $d = i\nu_{ei}/\omega$ . If  $d \ll 1$ , then the collisional term in  $\varepsilon_{xx}$  is imaginary one and of the order of  $(\nu_{ei}/\omega)(k_z^2 v_{Te}^2/\omega^2) \ll 1$  in comparison with the term  $a_{xx}$ . In the case  $d \gg 1$ , this collisional term becomes the real one and is  $\sim (k_z^2 v_{Te}^2/\omega^2) \ll 1$  in comparison with  $a_{xx}$ . Therefore, the collisional term in  $\varepsilon_{xx}$  is not taken into consideration in Equation (44). Then, the dispersion relation takes the form,

$$\omega^2 = k^2 c_A^2 + k_y^2 \frac{1}{m_i D} [D (T_{e0} + T_{i0}) + i\omega (G_1 + G_2 + G_3 + G_4)]. \quad (45)$$

The values  $D$  and  $G_i$ ,  $i = 1, 2, 3, 4$ , are given by expressions (A16)-(A20), where  $\partial/\partial t$  should be replaced by  $-i\omega$ . Calculating expression in the square brackets in Equation (45), we obtain

$$\omega^2 = k^2 c_A^2 + k_y^2 \frac{R}{m_i V}. \quad (46)$$

Here,

$$\begin{aligned} R = & T_{e0} (-i\gamma\omega + \Omega_\chi + \Omega_{Te} - \Omega_{ne}) (-i\omega + \Omega_{Ti} + 2\Omega_{ie}) \\ & + T_{i0} (-i\gamma\omega + \Omega_{Ti} - \Omega_{ni}) (-i\omega + \Omega_\chi + \Omega_{Te} + 2\Omega_{ei}) \\ & + (T_{e0} - T_{i0}) (-i\omega + \Omega_\chi + \Omega_{Te}) \Omega_{ie} + (T_{i0} - T_{e0}) (-i\omega + \Omega_{Ti}) \Omega_{ei} \\ & + T_{e0} (-i\omega + \Omega_{Ti}) \Omega_{Tei} + T_{e0} [-i(\gamma - 1)\omega - \Omega_{ne}] \Omega_{Tie} + T_{i0} (-i\gamma\omega + \Omega_{Ti} - \Omega_{ni}) \Omega_{Tei} \end{aligned} \quad (47)$$

and

$$V = [(-i\omega + \Omega_\chi + \Omega_{Te}) (-i\omega + \Omega_{Ti} + \Omega_{ie}) + (-i\omega + \Omega_{Ti}) (\Omega_{ei} + \Omega_{Tei})], \quad (48)$$

where  $\Omega_\chi = (\gamma - 1)(\chi_{e0}/n_{e0})k_z^2$ . All the values  $\Omega$  are defined by a system (14). We see that the first four terms on the right hand-side of expression (47) are symmetric ones relatively to contribution of electrons and ions. The last three terms are connected with perturbation of collision frequency  $\nu_{ei,ie}^\varepsilon(n_{i,e0}, T_e)$  because of the electron temperature perturbation. All the terms proportional to  $\Omega_{ei,ie}$  and  $\Omega_{Tei,ie}$  in expressions (47) and (48) are connected with the energy exchange in thermal equations (3) and (4). We see that this effect in a general case when  $T_{e0} \neq T_{i0}$  results in considerable modification (complication) of dispersion relation.

Nevertheless, it must be taken into account because the absence of thermal equilibrium between electrons and ions can be observed, for example, in the outer part of galaxy clusters (e.g., Markevitch et al. 1996; Fox & Loeb 1997; Ettori & Fabian 1998; Takizawa 1998).

Dispersion relation (46) has a general form which permits us to investigate analytically different limiting cases. When the thermal pressure is larger than the magnetic pressure, the case which is satisfied in the intracluster medium, one can omit the first term on the right hand-side of Equation (46). Then, we can write this equation in the form

$$\frac{\omega^2}{k_y^2 c_s^2} = \frac{R}{(T_{e0} + T_{i0}) V}, \quad (49)$$

where  $c_s$  is the sound speed. It is followed from Equation (49) that in the case  $\omega^2 \gg k_y^2 c_s^2$  or  $R \gg (T_{e0} + T_{i0}) V$ , dispersion relation is given by

$$V = 0. \quad (50)$$

In the opposite case,  $\omega^2 \ll k_y^2 c_s^2$  or  $R \ll (T_{e0} + T_{i0}) V$ , we have

$$R = 0. \quad (51)$$

### 9.1. SOLUTION OF DISPERSION RELATION $V = 0$

When  $\Omega_{ie,ei} = 0$ , i.e. at the absence of energy exchange, Equation (50) gives

$$(-i\omega + \Omega_\chi + \Omega_{Te}) (-i\omega + \Omega_{Ti}) = 0. \quad (52)$$

Thus, an instability can be generated by the electrons or ions. We see that Equation (52) has solutions which correspond to the isochoric ones in the MHD (Parker 1953; Field 1965).

When  $\Omega_{ie,ei} \rightarrow \infty$ , the case of a strong energy coupling, we obtain

$$\omega = -i \frac{(\Omega_\chi + \Omega_{Te}) \Omega_{ie} + \Omega_{Ti} (\Omega_{ei} + \Omega_{Tei})}{(\Omega_{ie} + \Omega_{ei} + \Omega_{Tei})}. \quad (53)$$

Taking into account that  $\Omega_{ie} = \Omega_{ei}$  at  $n_{e0} = n_{i0}$  and  $\Omega_{Tei} = -3\Omega_{ei}(T_{e0} - T_{i0})/2T_{e0}$  (Braginskii 1965), this equation becomes the following:

$$\omega = -i \frac{2\Omega_\chi + 2\Omega_{Te} + \Omega_{Ti}(3T_{i0}/T_{e0} - 1)}{1 + 3T_{i0}/T_{e0}}. \quad (54)$$

We see that if  $3T_{i0}/T_{e0} \ll 1$ , then  $\omega = -i(2\Omega_\chi + 2\Omega_{Te} - \Omega_{Ti})$ . Thus, the ions can contribute to instability when  $\Omega_{Ti} > 0$ . In this case, condition of instability takes the form

$$-\frac{\partial\Lambda_e(T_{e0})}{\partial T_{e0}} + \frac{\partial\Lambda_i(T_{i0})}{2\partial T_{i0}} > \frac{\chi_{e0}}{n_0^2} k_z^2,$$

where we have only taken into account cooling functions of electrons and ions (see Section 2). We see from this condition that instability can also be possible in the electron temperature domain where  $\partial\Lambda_e(T_{e0})/\partial T_{e0} > 0$ .

## 9.2. SOLUTION OF DISPERSION RELATION $R = 0$

We now consider the dispersion relation (51) which is appropriate in the case  $\omega^2 \ll k_y^2 c_s^2$ . This equation coincides with the dispersion relation in the fast sound speed regime (Nekrasov 2011). For reading convenience, we repeat here results given in the last paper. By using the temperature dependence of  $\nu_{ei,ie}^\varepsilon \sim T_e^{-3/2}$ , Equation (51) can be rewritten in the form,

$$\begin{aligned} & T_{e0}(-i\gamma\omega + \Omega_\chi + \Omega_{Te} - \Omega_{ne})(-i\omega + \Omega_{Ti} + 2\Omega_{ie}) \\ & + T_{i0}(-i\gamma\omega + \Omega_{Ti} - \Omega_{ni})(-i\omega + \Omega_\chi + \Omega_{Te} + 2\Omega_{ie}) \\ & + \Omega_{ie}(T_{e0} - T_{i0})(\Omega_\chi + \Omega_{Te} - \Omega_{Ti}) \\ & - \frac{3}{2}\Omega_{ie}(T_{e0} - T_{i0}) \left[ (-i\gamma\omega + \Omega_{Ti}) \left( 1 + \frac{T_{i0}}{T_{e0}} \right) - \Omega_{ne} - \frac{T_{i0}}{T_{e0}}\Omega_{ni} \right] = 0. \end{aligned} \quad (55)$$

The different limiting cases of Equation (55) are given in subsections of 9.2.

9.2.1. The case  $\Omega_{ie} = 0$

If we do not take into account the energy exchange,  $\Omega_{ie} = 0$ , and set  $T_{e0} = T_{i0}$ , then we obtain equation

$$\begin{aligned} 2\gamma\omega^2 + i[(\gamma + 1)(\Omega_\chi + \Omega_{Te} + \Omega_{Ti}) - \Omega_{ne} - \Omega_{ni}]\omega \\ - 2(\Omega_\chi + \Omega_{Te})\Omega_{Ti} + \Omega_{ne}\Omega_{Ti} + \Omega_{ni}(\Omega_\chi + \Omega_{Te}) = 0. \end{aligned} \quad (56)$$

Neglecting the contribution of the ion cooling and heating,  $\Omega_{Ti} = \Omega_{ni} = 0$ , we have

$$\omega = -\frac{i}{2\gamma}[(\gamma + 1)(\Omega_\chi + \Omega_{Te}) - \Omega_{ne}].$$

It is easy to see that this solution is a mixture of isochoric and isobaric solutions (Parker 1953; Field 1965) because we have taken into account the ion temperature perturbation. If we neglect the latter, i.e. neglect the second term  $\sim T_{i0}$  in Equation (55), we obtain the usual isobaric solution

$$\omega = -\frac{i}{\gamma}(\Omega_\chi + \Omega_{Te} - \Omega_{ne}).$$

We also see from Equation (55) that for short-wavelength perturbations when  $\Omega_\chi \gg \omega, \Omega_{Te}, \Omega_{ne}$  the thermal instability can arise due to the ion cooling function

$$\omega = -\frac{i}{T_{e0} + \gamma T_{i0}}[(T_{e0} + T_{i0})\Omega_{Ti} - T_{i0}\Omega_{ni}].$$

9.2.2. The case  $\Omega_{ie} \rightarrow \infty$

When the frequency  $\Omega_{ie}$  is much larger than other frequencies,  $2\Omega_{ie} \gg \omega, \Omega_\chi, \Omega_{Te,i}$ , and  $T_{e0} = T_{i0}$ , then the dispersion relation becomes the following:

$$\omega = -\frac{i}{2\gamma}(\Omega_\chi + \Omega_{Te} - \Omega_{ne} + \Omega_{Ti} - \Omega_{ni}). \quad (57)$$

This is isobaric solution with the electron and ion cooling-heating.

For different temperatures of electrons and ions,  $T_{e0} \neq T_{i0}$ , we obtain

$$\begin{aligned}
 i\frac{\gamma}{2} \left[ T_{e0} + \left( 4 + 3\frac{T_{i0}}{T_{e0}} \right) T_{i0} \right] \omega &= (3T_{e0} - T_{i0}) (\Omega_\chi + \Omega_{Te}) - \left[ \frac{5}{2}T_{e0} - 3T_{i0} \left( 1 + \frac{T_{i0}}{2T_{e0}} \right) \right] \Omega_{Ti} \\
 &\quad - \frac{1}{2} (3T_{i0} + T_{e0}) \left( \Omega_{ne} + \frac{T_{i0}}{T_{e0}} \Omega_{ni} \right).
 \end{aligned} \tag{58}$$

In the case  $T_{e0} \gg T_{i0}$ , this equation takes the form,

$$\omega = -\frac{i}{\gamma} [6 (\Omega_\chi + \Omega_{Te}) - \Omega_{ne} - 5\Omega_{Ti}].$$

In the opposite case,  $T_{e0} \ll T_{i0}$ , we obtain the ion isobaric solution

$$\omega = -\frac{i}{\gamma} (\Omega_{Ti} - \Omega_{ni}).$$

### 9.2.3. General case

In a general case, Equation (55) can be written in the form

$$g_0 \omega^2 + i g_1 \omega - g_2 = 0, \tag{59}$$

where

$$\begin{aligned}
g_0 &= \gamma (T_{e0} + T_{i0}), \\
g_1 &= [(\gamma T_{i0} + T_{e0}) (\Omega_\chi + \Omega_{Te}) + (\gamma T_{e0} + T_{i0}) \Omega_{Ti} - T_{e0} \Omega_{ne} - T_{i0} \Omega_{ni}] \\
&\quad + \frac{1}{2} \gamma \left[ T_{e0} + T_{i0} \left( 4 + 3 \frac{T_{i0}}{T_{e0}} \right) \right] \Omega_{ie}, \\
g_2 &= T_{e0} (\Omega_\chi + \Omega_{Te} - \Omega_{ne}) \Omega_{Ti} + T_{i0} (\Omega_{Ti} - \Omega_{ni}) (\Omega_\chi + \Omega_{Te}) \\
&\quad + (3T_{e0} - T_{i0}) \Omega_{ie} (\Omega_\chi + \Omega_{Te}) - \left[ \frac{5}{2} T_{e0} - 3T_{i0} \left( 1 + \frac{T_{i0}}{2T_{e0}} \right) \right] \Omega_{ie} \Omega_{Ti} \\
&\quad - \frac{1}{2} (T_{e0} + 3T_{i0}) \Omega_{ie} \left( \Omega_{ne} + \Omega_{ni} \frac{T_{i0}}{T_{e0}} \right).
\end{aligned} \tag{60}$$

Equation (59) can be solved numerically for known cooling-heating functions and temperatures.

## 10. DISCUSSION

Applying the operator  $\nabla \cdot$  to Equation (8), we can see that  $n_{e1} = n_{i1}$ , if the electron and ion number densities are only perturbed. However, in our general calculations in the Appendix, the values  $\nabla \cdot \mathbf{v}_{e1}$  and  $\nabla \cdot \mathbf{v}_{i1}$  are considered as different in the cases  $n_{e0} \neq n_{i0}$  and  $n_{e0} = n_{i0}$ . Solving equations of motion, we do not need to use the condition  $n_{i1} = n_{e1}$ . These equations permit us to treat a general case  $n_{i1} \neq n_{e1}$  that can be appropriate for multicomponent plasmas. To derive the dispersion relation, we find expressions for perturbed velocities and calculate the perturbed current which then is used in Equation (8) in his given form. The perturbed current does not contain density perturbations at the absent of background flow velocities. Equations (24)-(29) are justified under conditions (18) and (19) and can be applied for both  $n_{i0} = n_{e0}$ ,  $n_{i1} = n_{e1}$  and  $n_{i0} \neq n_{e0}$ ,  $n_{i1} \neq n_{e1}$  (for multicomponent plasmas) cases. Taking into account condition (18), we obtain from Equations (24), (25), (28), and (29) that  $\nabla \cdot \mathbf{v}_{i1} = \nabla \cdot \mathbf{v}_{e1} = -(c/B_0)(\partial E_{1x}/\partial y)$  for

perturbations (44).

From the results obtained above, we can estimate relative perturbations of number density and pressure in the fast dynamical or long-wavelength regime (18). Using, for example, Equations (2), (24) and (28) and keeping main terms, we find equation for the ion density perturbation,

$$\frac{\partial n_{i1}}{n_{i0} \partial t} = \frac{1}{\omega_{ci}} \frac{\partial F_{i1x}}{\partial y}. \quad (61)$$

From Equation (21), it follows that

$$P_{i1} = (-\lambda_i + \mu_i) \frac{1}{\omega_{ci}} \frac{\partial^2 F_{i1x}}{\partial y \partial t}, \quad (62)$$

where we have used the relation  $m_i \mathbf{F}_{i1} = -m_e \mathbf{F}_{e1}$ . The value  $P_{j1}$  is connected with the pressure perturbation  $p_{j1}$  as follows

$$P_{j1} = -\frac{1}{m_j n_{j0}} \frac{\partial p_{j1}}{\partial t}. \quad (63)$$

From Equations (61)-(63), we obtain

$$\frac{\partial n_{i1}}{n_{i0} \partial t} = \frac{T_{i0}}{(\lambda_i - \mu_i) m_i} \frac{p_{i1}}{p_{i0}}. \quad (64)$$

Analogously, we have for electrons

$$\frac{\partial n_{e1}}{n_{e0} \partial t} = \frac{T_{e0}}{(\lambda_e - \mu_e) m_e} \frac{p_{e1}}{p_{e0}}, \quad (65)$$

where  $n_{e1} = n_{i1}$ . Thus,

$$\frac{p_{i1}}{(\lambda_i - \mu_i) m_i} = \frac{p_{e1}}{(\lambda_e - \mu_e) m_e}.$$

Taking into account notations (23), we see that  $n_{i,e1}/n_{i0} \sim p_{i,e1}/p_{e0}$  and  $p_{i1}/p_{i0} \sim p_{e1}/p_{e0}$ . Using the dispersion relation (44) in the case of neglect the magnetic field, we obtain from Equations (64) and (65) equation connecting the sum of pressures and density perturbation,

$$\frac{\partial^2 n_{i1}}{\partial t^2} = \frac{1}{m_i} \frac{\partial^2}{\partial y^2} (p_{e1} + p_{i1}).$$



It is easy to see from Equations (33), (38), and (44) that Equation (51) corresponds to isobaric regime where the sum of electron and ion pressure perturbations is smaller than electron or ion pressure perturbation (see also Nekrasov 2011). The perturbation of number density is due to electric drift (see Equation (61)).

Dispersion relation (50) is satisfied in the case  $\omega^2 \gg k_y^2 c_s^2$ . Taking into account condition (18), unstable perturbations have  $k_y^2 \lesssim (m_i/m_e) k_z^2$ . Thus, the transverse wavelength  $\lambda_\perp \gtrsim (m_e/m_i)^{1/2} \lambda_z$  and can be both less and larger than the longitudinal wavelength  $\lambda_z$ . From other side, in the case  $\omega^2 \ll k_y^2 c_s^2$ , dispersion relation (51) describes unstable perturbations strongly elongated along the magnetic field,  $k_y^2 \gg (m_i/m_e) k_z^2$  or  $\lambda_z \gg (m_i/m_e)^{1/2} \lambda_\perp$ . In this case, very thin filaments are generated. Thus, a wide spectrum of wavelengths of perturbations along and across the magnetic field can be formed in the framework of conditions (18), (19), (34), and (41).

A general form of dispersion relation (46) including the thermal exchange and different temperatures allows us to consider various cases, which can be realized in real situations. We can investigate a weakly, strongly, and intermediate thermal coupling (see Sections 5.5 and 5.6). In particular, Equations (52) and (56) are available for a weak thermal coupling, while Equations (53), (54), (57), and (58) are appropriate in the case of strong coupling. The intermediate case is described by Equation (59), where coefficients (60) contain both different temperatures and different cooling functions.

We have shown that unstable perturbations have an electromagnetic nature (see Equation (44)). Thus, a consideration of only potential perturbations is in general not adequate.

## 11. ASTROPHYSICAL IMPLICATIONS

We shortly outline some important points of our investigation for possible observations. We have found growth rates, which contain in the clear form the separate contribution of cooling functions of electrons and ions. It is obvious that both components (in multicomponent media also dust grains, neutrals, and so on) can result in thermal instability in the same extent. This fact considerably extends possibilities for the medium to become unstable. In this connection, it is important to know the functional dependence of cooling functions on the temperature and density for each species. Unfortunately, at present, there is not sufficient information on this subject in astrophysical literature. The range of scale lengths of unstable perturbations can enlarge due to contribution to instability of other species except electrons. For example, short-wavelength perturbations, which must be stable because of a large electron thermal conduction, can be unstable due to contribution of ions to cooling of medium (see Section 9). In the long-wavelength regime (18), scale sizes of unstable perturbations across the magnetic field can have a wide spectrum and be, in particular, very elongated along the magnetic field. Such filaments are observed in galaxy clusters (e.g., Conselice et al. 2001; Salomé et al. 2006) and in the solar corona (e.g., Tandberg-Hanssen 1974; Karpen et al. 1989). Different temperatures of electrons and ions assumed in this paper can be observed in galaxy clusters (Markevitch et al. 1996; Fox & Loeb 1997; Ettori & Fabian 1998; Takizawa 1998). This fact proves that dynamical and statistical processes could have timescales of the same order. It is clear that real situations in astrophysical objects are much more complicated to be captured by simplified theories. Knowledge of fundamental processes and more detailed conditions from observations are very important for theoretical models and, in particular, for further investigation of thermal instabilities.

## 12. CONCLUSION

We have studied thermal instability in the electron-ion magnetized plasma which is relevant to galaxy clusters, solar corona, and other two-component astrophysical objects. The multicomponent plasma approach have been applied to derive the dispersion relation for the condensation mode in the case in which the dynamical time is smaller than a time the particles need to cover the wavelength of perturbations along the magnetic field due to their thermal velocity. Our dispersion relation takes into account the electron and ion cooling-heating functions, collisions in momentum equations, energy exchange in thermal equations, different background temperatures of electrons and ions, and perturbation of energy exchange collision frequency due to density and temperature perturbations. Different limiting cases of dispersion relation have been considered and simple expressions for growth rates have been obtained. We have shown that perturbations have an electromagnetic nature. We have found that at conditions under consideration transverse scale sizes of unstable perturbations can have a wide spectrum relatively to longitudinal scale sizes and, in particular, form very thin filaments. General expressions for dynamical variables obtained in this paper can be applied for astrophysical and laboratory plasmas also containing the neutrals, dust grains, and other species. The results obtained can be useful for interpretation of observations of dense cold regions in astrophysical objects.

In this paper, we have investigated the linear stage of thermal instability in the multicomponent medium. The instability development can result in plasma turbulence when transport coefficients become dependent not on Coulomb collisions but on the energy of turbulence. In this case, a nonlinear consideration of a problem is necessary.

### 13. ACKNOWLEDGMENTS

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## A. APPENDIX

### A.1. Perturbed velocities of species

In the linear approximation, Equation (1) for the perturbed velocity  $\mathbf{v}_{j1}$  takes the form

$$\frac{\partial \mathbf{v}_{j1}}{\partial t} = -\frac{\nabla p_{j1}}{m_j n_{j0}} + \mathbf{F}_{j1} + \frac{q_j}{m_j c} \mathbf{v}_{j1} \times \mathbf{B}_0, \quad (\text{A1})$$

where  $p_{j1} = n_{j0}T_{j1} + n_{j1}T_{j0}$ . From this equation, we can find solutions for the components of  $\mathbf{v}_{j1}$ . For simplicity, we assume that  $\partial/\partial x = 0$  because a system is symmetric in the transverse direction relative to the  $z$ -axis. Then, the  $x$ -component of Equation (A1) gives

$$\frac{\partial v_{j1x}}{\partial t} = F_{j1x} + \omega_{cj} v_{j1y}, \quad (\text{A2})$$

where  $\omega_{cj} = q_j B_0 / m_j c$  is the cyclotron frequency. Differentiating Equation (A1) over  $t$  and using Equation (2) in the linear approximation and Equations (15), (16), and (A2), we obtain for the  $y$ -component of Equation (A1)

$$\left( \frac{\partial^2}{\partial t^2} + \omega_{cj}^2 \right) v_{j1y} = \frac{\partial P_{j1}}{\partial y} + Q_{j1y}, \quad (\text{A3})$$

where

$$\begin{aligned} P_{e1} &= -\frac{G_2}{Dm_e} \frac{\partial}{\partial t} \nabla \cdot \mathbf{v}_{i1} + \left( \frac{T_{e0}}{m_e} - \frac{G_1}{Dm_e} \frac{\partial}{\partial t} \right) \nabla \cdot \mathbf{v}_{e1}, \\ P_{i1} &= -\frac{G_3}{Dm_i} \frac{\partial}{\partial t} \nabla \cdot \mathbf{v}_{e1} + \left( \frac{T_{i0}}{m_i} - \frac{G_4}{Dm_i} \frac{\partial}{\partial t} \right) \nabla \cdot \mathbf{v}_{i1}. \end{aligned} \quad (\text{A4})$$

The value  $P_{j1}$  is connected with the pressure perturbation (see Equation (A1)). Using Equations (A2) and (A3), we find

$$\frac{\partial}{\omega_{cj}\partial t} \left[ \left( \frac{\partial^2}{\partial t^2} + \omega_{cj}^2 \right) v_{j1x} - Q_{j1x} \right] = \frac{\partial P_{j1}}{\partial y} \quad (\text{A5})$$

In Equations (A3) and (A5), notations

$$\begin{aligned} Q_{j1y} &= -\omega_{cj} F_{j1x} + \frac{\partial F_{j1y}}{\partial t}, \\ Q_{j1x} &= \omega_{cj} F_{j1y} + \frac{\partial F_{j1x}}{\partial t} \end{aligned} \quad (\text{A6})$$

are introduced. We see from these equations that the thermal pressure effect on the velocity  $v_{i1x}$  is much larger than that on  $v_{i1y}$  when  $\partial/\partial t \ll \omega_{ci}$ . The  $z$ -component of Equation (A1) can be written in the form

$$\frac{\partial^2 v_{j1z}}{\partial t^2} = \frac{\partial P_{j1}}{\partial z} + \frac{\partial F_{j1z}}{\partial t}. \quad (\text{A7})$$

## A.2. Calculation of $\nabla \cdot \mathbf{v}_{j1}$ and $P_{j1}$

We have

$$\nabla \cdot \mathbf{v}_{j1} = \frac{\partial v_{j1y}}{\partial y} + \frac{\partial v_{j1z}}{\partial z}. \quad (\text{A8})$$

Using Equations (A3), (A4), (A7), and (A8), we obtain

$$\begin{aligned} L_{1e} \nabla \cdot \mathbf{v}_{e1} + L_{2e} \nabla \cdot \mathbf{v}_{i1} &= H_{e1}, \\ L_{1i} \nabla \cdot \mathbf{v}_{i1} + L_{2i} \nabla \cdot \mathbf{v}_{e1} &= H_{i1}. \end{aligned} \quad (\text{A9})$$

Here

$$H_{j1} = \frac{\partial^3 Q_{j1y}}{\partial y \partial t^2} + \left( \frac{\partial^2}{\partial t^2} + \omega_{cj}^2 \right) \frac{\partial^2 F_{j1z}}{\partial z \partial t} \quad (\text{A10})$$

and operators  $L_{1j}$  and  $L_{2j}$  are the following:

$$\begin{aligned} L_{1e} &= \left( \frac{\partial^2}{\partial t^2} + \omega_{ce}^2 \right) \frac{\partial^2}{\partial t^2} - L_{3e} \left( \frac{T_{e0}}{m_e} - \frac{G_1}{Dm_e} \frac{\partial}{\partial t} \right), \\ L_{1i} &= \left( \frac{\partial^2}{\partial t^2} + \omega_{ci}^2 \right) \frac{\partial^2}{\partial t^2} - L_{3i} \left( \frac{T_{i0}}{m_i} - \frac{G_4}{Dm_i} \frac{\partial}{\partial t} \right), \\ L_{2e} &= L_{3e} \frac{G_2}{Dm_e} \frac{\partial}{\partial t}, L_{2i} = L_{3i} \frac{G_3}{Dm_i} \frac{\partial}{\partial t}, \\ L_{3j} &= \frac{\partial^4}{\partial y^2 \partial t^2} + \left( \frac{\partial^2}{\partial t^2} + \omega_{cj}^2 \right) \frac{\partial^2}{\partial z^2}. \end{aligned} \quad (\text{A11})$$

From a system of equations (A9), we find

$$L \nabla \cdot \mathbf{v}_{e1} = -L_{2e} H_{i1} + L_{1i} H_{e1}, \quad (\text{A12})$$

$$L \nabla \cdot \mathbf{v}_{i1} = -L_{2i} H_{e1} + L_{1e} H_{i1},$$

where

$$L = L_{1e} L_{1i} - L_{2e} L_{2i}. \quad (\text{A13})$$

The values  $P_{e1}$  and  $P_{i1}$  can be found, substituting solutions (A12) into expressions (A4),

$$\begin{aligned} LP_{i1} &= \left[ \frac{G_3}{Dm_i} \frac{\partial}{\partial t} L_{2e} + \left( \frac{T_{i0}}{m_i} - \frac{G_4}{Dm_i} \frac{\partial}{\partial t} \right) L_{1e} \right] H_{i1} \\ &\quad - \left[ \frac{G_3}{Dm_i} \frac{\partial}{\partial t} L_{1i} + \left( \frac{T_{i0}}{m_i} - \frac{G_4}{Dm_i} \frac{\partial}{\partial t} \right) L_{2i} \right] H_{e1}. \end{aligned} \quad (\text{A14})$$

$$\begin{aligned} LP_{e1} &= \left[ \frac{G_2}{Dm_e} \frac{\partial}{\partial t} L_{2i} + \left( \frac{T_{e0}}{m_e} - \frac{G_1}{Dm_e} \frac{\partial}{\partial t} \right) L_{1i} \right] H_{e1} \\ &\quad - \left[ \frac{G_2}{Dm_e} \frac{\partial}{\partial t} L_{1e} + \left( \frac{T_{e0}}{m_e} - \frac{G_1}{Dm_e} \frac{\partial}{\partial t} \right) L_{2e} \right] H_{i1}, \end{aligned} \quad (\text{A15})$$

### A.3. Expressions for $D$ and $G_{1,2,3,4}$

We now give expressions for values defined by a system (17):

$$D = \left( \frac{\partial}{\partial t} + \Omega_\chi + \Omega_{Te} \right) \left( \frac{\partial}{\partial t} + \Omega_{Ti} + \Omega_{ie} \right) \frac{\partial^2}{\partial t^2} + (\Omega_{ei} + \Omega_{Tei}) \left( \frac{\partial}{\partial t} + \Omega_{Ti} \right) \frac{\partial^2}{\partial t^2}, \quad (\text{A16})$$

$$G_1 = T_{e0} \left[ \Omega_{ne} - (\gamma - 1) \frac{\partial}{\partial t} \right] \left( \frac{\partial}{\partial t} + \Omega_{Ti} + \Omega_{ie} \right) \frac{\partial}{\partial t} + \Omega_{ei} (T_{e0} - T_{i0}) \left( \frac{\partial}{\partial t} + \Omega_{Ti} \right) \frac{\partial}{\partial t}, \quad (\text{A17})$$

$$G_2 = \Omega_{ei} T_{i0} \left[ \Omega_{ni} - (\gamma - 1) \frac{\partial}{\partial t} \right] \frac{\partial}{\partial t} + \Omega_{ei} (T_{e0} - T_{i0}) \left( \frac{\partial}{\partial t} + \Omega_{Ti} \right) \frac{\partial}{\partial t}, \quad (\text{A18})$$

$$G_3 = (\Omega_{Tie} + \Omega_{ie}) T_{e0} \left[ \Omega_{ne} - (\gamma - 1) \frac{\partial}{\partial t} \right] \frac{\partial}{\partial t} - \Omega_{ie} (T_{e0} - T_{i0}) \left( \frac{\partial}{\partial t} + \Omega_{\chi} + \Omega_{Te} \right) \frac{\partial}{\partial t}, \quad (\text{A19})$$

$$G_4 = T_{i0} \left( \frac{\partial}{\partial t} + \Omega_{\chi} + \Omega_{Te} + \Omega_{Tei} + \Omega_{ei} \right) \left[ \Omega_{ni} - (\gamma - 1) \frac{\partial}{\partial t} \right] \frac{\partial}{\partial t} - \Omega_{ie} (T_{e0} - T_{i0}) \left( \frac{\partial}{\partial t} + \Omega_{\chi} + \Omega_{Te} \right) \frac{\partial}{\partial t}. \quad (\text{A20})$$

#### A.4. Simplification of Equations (A14) and (A15)

We further calculate coefficients by  $H_{j1}$  in Equations (A14) and (A15). Using expressions (A11), we find

$$\frac{G_3}{Dm_i} \frac{\partial}{\partial t} L_{1i} + \left( \frac{T_{i0}}{m_i} - \frac{G_4}{Dm_i} \frac{\partial}{\partial t} \right) L_{2i} = \frac{G_3}{Dm_i} \left( \frac{\partial^2}{\partial t^2} + \omega_{ci}^2 \right) \frac{\partial^3}{\partial t^3} \quad (\text{A21})$$

and

$$\frac{G_3}{Dm_i} \frac{\partial}{\partial t} L_{2e} + \left( \frac{T_{i0}}{m_i} - \frac{G_4}{Dm_i} \frac{\partial}{\partial t} \right) L_{1e} = \frac{1}{D} \left( D \frac{T_{i0}}{m_i} - \frac{G_4}{m_i} \frac{\partial}{\partial t} \right) \left( \frac{\partial^2}{\partial t^2} + \omega_{ce}^2 \right) \frac{\partial^2}{\partial t^2} + \frac{1}{Dm_e m_i} L_{3e} K. \quad (\text{A22})$$

In Equation (A22), we have introduced notation

$$K = \frac{1}{D} (G_2 G_3 - G_1 G_4) \frac{\partial^2}{\partial t^2} + (T_{e0} G_4 + T_{i0} G_1) \frac{\partial}{\partial t} - D T_{e0} T_{i0}. \quad (\text{A23})$$

Calculations show that the value  $(G_2 G_3 - G_1 G_4)$  has a simple form, i.e.,

$$\frac{1}{D} (G_2 G_3 - G_1 G_4) = \Omega_{ie} (T_{e0} - T_{i0}) T_{e0} \left[ \Omega_{ne} - (\gamma - 1) \frac{\partial}{\partial t} \right] + \Omega_{ei} (T_{i0} - T_{e0}) T_{i0} \left[ \Omega_{ni} - (\gamma - 1) \frac{\partial}{\partial t} \right] \quad (\text{A24})$$

$$- T_{e0} T_{i0} \left[ \Omega_{ne} - (\gamma - 1) \frac{\partial}{\partial t} \right] \left[ \Omega_{ni} - (\gamma - 1) \frac{\partial}{\partial t} \right].$$

Using expressions (A16), (A17), (A20), and (A24), we obtain for the operator  $K$  (A23) the simple form,

$$K = -\Omega_{ie}T_{e0}W_e\frac{\partial^2}{\partial t^2} - (\Omega_{ei}T_{i0} + \Omega_{Tei}T_{e0})T_{i0}W_i\frac{\partial^2}{\partial t^2} - T_{e0}T_{i0}W_eW_i\frac{\partial^2}{\partial t^2}, \quad (\text{A25})$$

where notations

$$\begin{aligned} W_e &= \gamma \frac{\partial}{\partial t} + \Omega_\chi + \Omega_{Te} - \Omega_{ne}, \\ W_i &= \gamma \frac{\partial}{\partial t} + \Omega_{Ti} - \Omega_{ni} \end{aligned} \quad (\text{A26})$$

are introduced. Using Equations (A21) and (A22), Equation (A14) for  $P_{i1}$  takes the form,

$$\begin{aligned} DLP_{i1} &= \left[ \left( D\frac{T_{i0}}{m_i} - \frac{G_4}{m_i}\frac{\partial}{\partial t} \right) \left( \frac{\partial^2}{\partial t^2} + \omega_{ce}^2 \right) \frac{\partial^2}{\partial t^2} + \frac{1}{m_em_i}L_{3e}K \right] H_{i1} \\ &\quad - \frac{G_3}{m_i} \left( \frac{\partial^2}{\partial t^2} + \omega_{ci}^2 \right) \frac{\partial^3}{\partial t^3} H_{e1}. \end{aligned} \quad (\text{A27})$$

Analogous consideration of Equation (A15) leads to the following equation for  $P_{e1}$ :

$$\begin{aligned} DLP_{e1} &= \left[ \left( D\frac{T_{e0}}{m_e} - \frac{G_1}{m_e}\frac{\partial}{\partial t} \right) \left( \frac{\partial^2}{\partial t^2} + \omega_{ci}^2 \right) \frac{\partial^2}{\partial t^2} + \frac{1}{m_im_e}L_{3i}K \right] H_{e1} \\ &\quad - \frac{G_2}{m_e} \left( \frac{\partial^2}{\partial t^2} + \omega_{ce}^2 \right) \frac{\partial^3}{\partial t^3} H_{i1}. \end{aligned} \quad (\text{A28})$$

Operators

$$\begin{aligned} D\frac{T_{e0}}{m_e} - \frac{G_1}{m_e}\frac{\partial}{\partial t}, \\ D\frac{T_{i0}}{m_i} - \frac{G_4}{m_i}\frac{\partial}{\partial t} \end{aligned}$$

can be found by using Equations (A16), (A17), (A20), and (A26)

$$\begin{aligned} D\frac{T_{e0}}{m_e} - \frac{G_1}{m_e}\frac{\partial}{\partial t} &= \frac{T_{e0}}{m_e}W_e \left( \frac{\partial}{\partial t} + \Omega_{Ti} + \Omega_{ie} \right) \frac{\partial^2}{\partial t^2} \\ &\quad + \frac{1}{m_e}(T_{e0}\Omega_{Tei} + \Omega_{ei}T_{i0}) \left( \frac{\partial}{\partial t} + \Omega_{Ti} \right) \frac{\partial^2}{\partial t^2}, \\ D\frac{T_{i0}}{m_i} - \frac{G_4}{m_i}\frac{\partial}{\partial t} &= \frac{T_{i0}}{m_i}W_i \left( \frac{\partial}{\partial t} + \Omega_\chi + \Omega_{Te} + \Omega_{ei} + \Omega_{Tei} \right) \frac{\partial^2}{\partial t^2} \\ &\quad + \frac{T_{e0}}{m_i}\Omega_{ie} \left( \frac{\partial}{\partial t} + \Omega_\chi + \Omega_{Te} \right) \frac{\partial^2}{\partial t^2}. \end{aligned} \quad (\text{A29})$$



### A.5. Operator $L$ in a general form

Using expressions (A11), we find from Equation (A13)

$$L = M - N - \frac{1}{m_e m_i D} L_{3e} L_{3i} K, \quad (\text{A30})$$

where

$$\begin{aligned} M &= \left( \frac{\partial^2}{\partial t^2} + \omega_{ce}^2 \right) \left( \frac{\partial^2}{\partial t^2} + \omega_{ci}^2 \right) \frac{\partial^4}{\partial t^4}, \\ N &= \left( \frac{\partial^2}{\partial t^2} + \omega_{ci}^2 \right) \frac{\partial^2}{\partial t^2} L_{3e} \left( \frac{T_{e0}}{m_e} - \frac{G_1}{D m_e} \frac{\partial}{\partial t} \right) + \left( \frac{\partial^2}{\partial t^2} + \omega_{ce}^2 \right) \frac{\partial^2}{\partial t^2} L_{3i} \left( \frac{T_{i0}}{m_i} - \frac{G_4}{D m_i} \frac{\partial}{\partial t} \right), \end{aligned} \quad (\text{A31})$$

and  $K$  is defined by Equation (A25).

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